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Living on the Edge of the Catastrophe

by

Andrea Rampa and Alessio D'Amato

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Living on the Edge of the Catastrophe*

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Abstract

The aim of this paper is to provide a theoretical model in order to analyse environmental policy under uncertainty regarding the possibility of a natural disaster. We adopt a two-periods analytical model, to investigate two different institutional settings, one featuring a myopic social planner, choosing emissions in each time period to maximize current net benefits, and one featuring a forward-looking planner, who maximizes the expected net present value of welfare across the two periods. As in Barrett (2013), uncertainty regards a threshold pollution level that, if violated, triggers a natural disaster. We conclude that under a myopic social planner welfare may increase or decrease over time, while in a non-myopic scenario welfare always increases across periods. Also, our model supports the idea that a myopic social planner pushes emissions closer to the edge of the natural disaster, but then, if the latter does not take place in the first period, benefits from having done that in terms of welfare in the second period. Introducing a stochastic decay rate, we also show that the environment may reward (punish) myopic behaviour ex post. Finally, the comparison between myopic and forward looking settings is not straightforward: this depends on a risk spreading vs. information learning trade off.

Keywords: Catastrophe, Uncertainty, Environmental Policy, Risk, Natural Disaster.

JEL Classification: Q54, O13, Q38

1 Introduction

Uncertainty is an unavoidable issue related to several environmental problems. It is therefore the subject of a significant amount of literature, starting from the seminal contributions by Weitzman (1974), Robert and Spence (1976) and many others. In his contribution on price vs. quantities, Weitzman (1974) was the first to compare environmental policy instruments in the presence of uncertainty, showing that uncertainty on the benefits side does not seem to matter in environmental policy design: only the uncertainty on the costs side matters. More recent papers, as among others Stavins (1996), show instead that "refining" Weitzman model can lead to significantly different outcomes in terms of socially desirable policy design; more specifically, Stavins (1996) asserts that uncertainty on the benefits side can indeed trigger a social loss, if cost and benefit uncertainty is correlated.

In many studies, uncertainty affects the amount of damage that the emissions cause; Tarui and Polasky

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(2005) work, is an example of this. In their paper, the authors introduce a multi period environmental policy model under uncertainty and investigate the socially desirable environmental policy design assuming that the policy makers can learn new information about environmental damages between different periods. In such a way, uncertainty plays a role not only in the chosen instrument, but also in the chosen timing of the policy. More precisely, Tarui and Polasky (2005) provide a game-theoretic model of pollution regulation with endogenous technology adoption and learning involving a policymaker and a polluting firm. The authors compare taxes and standard policy under discretion or rules and they develop a welfare analysis. The timing of intervention, rules or discretion, affects the acquisition of new information by regulator. Under discretion, the regulator resolves the uncertainty before designing the policy, instead, under rules, the regulator designs the policy before resolving the uncertainty.

We depart from the above cited literature by modelling a two periods regulatory setting where an environmental authority chooses the strictness of environmental policy. The environmental authority can be myopic (maximizing social welfare in each time period) or forward looking (choosing emissions in the first period accounting for all the consequences of her choices on both periods). In both cases, the strictness of environmental policy in the first and second periods is chosen under uncertainty; on the other hand, we take a rather extreme view of uncertainty, namely we model the possibility of a natural disaster to take place if a (stochastic) threshold is exceeded by pollution levels in any of the two periods. In such a way, our paper is close to Tarui and Polasky (2005) in terms of analytical structure. We however simplify it without accounting for technology adoption, while we keep the possibility of learning across the two periods. As we will clarify in the paper, modelling environmental damages as a natural disaster (of a yes or no character) allows us to investigate previously not assessed issues arising in choosing the optimal timing of environmental policy. Other papers related to our work are mostly linked to the role played by uncertainty (obviously) and to the modelling of natural disasters, including climate change.

Newell and Pizer (2003) analyse the role played by uncertainty in determining the costs of global climate change policy. Pollution is treated as a stock externality that involves uncertainty regarding its amount and its direct damage. Moreover, the authors compare the price and quantity based policies, then applying their results to climate change intervention. Costello and Karp (2004) entail uncertainty on the abatement cost function and focus on the possibility of learning and strategic behaviour by the regulated firm.

In the papers listed above, and in general in the literature, uncertainty usually affects costs, the damage function or both of them. Even those studies in which a catastrophic scenario is uncertain, at least one agent knows the causality between human activities and natural disasters. On the other hand, we focus on the case of uncertainty on the environmental damages side, but we work with an uncertain threshold level, that if violated, causes a catastrophic event.

There are very few examples of papers dealing with uncertainty about the magnitude of a threshold level triggering an environmental disaster. Barrett (2013), among the very few, analyses uncertainty by using both a uniform random variable and a Gaussian probability distribution function. On the other hand, Barrett's work is focused on the impact of the uncertain threshold on the (in)success of climate treaties.

In our paper, we borrow the uncertain threshold treatment from Barrett (2013), and analyse its consequences in terms of desirable policy design in a setting *a la* Tarui and Polasky (2005). We depart from Barrett (2013), as we do not focus on climate related disasters. Indeed, the simple way in which uncertainty is modelled in our paper makes it more suitable for more "local" environmental disasters, such as weather accidents, floods or toxic pollutants.

More specifically, we model a two periods setting featuring a firm generating emissions in each period and subject to regulation by an authority in charge of maximizing social welfare. As already outlined, the authority can be myopic or forward looking. In our stylized model, pollution generates linear benefits but also increases the level of emissions, that accumulate and possibly contribute to reach an uncertain threshold triggering an environmental disaster. The disaster can take place in the first period (in which case it generates a social loss for both periods, which adds to the loss of benefits in the second period) or in the second period (in which case it generates a social loss only in the same period). We limit our case to a very simple command and control policy, to keep matters as readable as possible. Also, to focus on our main issue, we disregard the environmental damages related to pollution which are not related to the risk of the natural disaster. Our model features a peculiar kind of learning; namely, the information obtained between the first and the second period if the environmental disaster does not take place is a positive function of the risk ran during the first period. In other words, the more I pollute in the first period, the closer I get to the uncertain threshold (which is bad), but the more I learn about the threshold (if the disaster does not take place) between the first and the second period (which is likely to improve welfare). This is a particular feature that is not present in the literature, but it is extremely close to reality. The idea is to model the incentive of the regulator and the firm to go as close as possible to the borderline of the catastrophe in order to discover where is the line between the disaster and safety. This implies an incentive to increase emissions in the first period that we label as *"dangerous learning"*, which is linked to the existence of an absorption capacity by the environment related to the pollutant at stake, and introduces an additional source of interest in the debate on the desirable timing of environmental policy.

The outline of the paper is as follows. Section 2 presents the basic structure of the model. Section 3 argues the nature of the uncertainty. Section 4 develops the model under myopic and non-myopic preference structure. Section 5 entails the comparison between the two preference structures. A numerical example is also presented in this section. Section 6 stresses the parameters of the model through a comparative statics. Section 7 adds to the model a stochastic learning and compares it with the basic model. Section 8 concludes.

2 The Model

We model a two periods setting (with $i = 1, 2$) where a social planner aims at maximizing a social welfare function which is composed by the benefits linked to emissions in each time period and by the social cost of a possible natural disaster. Thus, there is a benefit in each period $i = 1, 2$ and a social cost X that arises if the disaster takes place. Emissions are generated by a regulated firm featuring benefits $B(\cdot)$. We model the firm in the simplest possible way, by assuming that it wishes to emit any arbitrarily large level, so that the actual firm's emissions are given by the target e_i set in each period $i = 1, 2$ by the environmental authority.

The disaster could occur during the first period (in which case it generates damages in both periods), during the second one or never. The possible social loss X related to the natural disaster is associated with the probability $F(e_1)$ (i.e. the probability that the disaster occurs in the first period) or $F(e_1, e_2)$ (i.e. the probability that the disaster happens in the second period). The specific structure of the probability linked to the possible catastrophe is central to the paper and will be described in details in the next section. A regulated firm generates emissions.

The timing of intervention depends on the structure of the objective function of the environmental authority. We distinguish two cases: a myopic preference function and a non-myopic one, according to the related literature. Indeed, following Kurz (1987): *"Myopic decision rules refers to...* preferences depending *...only upon the values of the exogenous variable at time i, disregarding any information about future conditions of the economic environment"*.

Thus, in the case of myopic preferences there will be an objective function only depending on benefits and damages in each period $i = 1, 2$. This implies that in the first time period the regulator chooses e_1 to maximize: $[1 - F(e_1)]\, B(e_1) + [F(e_1)]\, [-X]$, whereas, during the second time period any variable regarding the first time period is treated as exogenous and the authority chooses e_2 to maximize: $[1 - F(e_1, e_2)]\, B(e_2) + [F(e_1, e_2)]\, [-X]$.

Instead, with a forward looking, or non-myopic, framework, emissions' decision are simultaneously taken at time $i = 1$, so that the environmental authority chooses e_1 and e_2 to maximize: $[1 - F(e_1)]\, B(e_1) + [1 - F(e_1)]\, [1 - F(e_1, e_2)]\, B(e_2) + [F(e_1)] \left[-X - X(1 + \delta)^{-1}\right] + [1 - F(e_1)]\, [F(e_1, e_2)] \left[-X(1 + \delta)^{-1}\right]$.

Straightforward, with non-myopic preference, we have to consider several cases: the case in which the disaster does not occur during the first period (probability $[1 - F(e_1)]$); the case in which the disaster occurs during the first period (with probability $F(e_1)$) and then it generates damages along the two time periods, the corresponding present value of damages being $-X - X(1+\delta)^{-1}$, where δ is the discount rate; the case in which the disaster does not occur during the first time period but it occurs during the second one (probability: $[1 - F(e_1)]\, F(e_1, e_2)$) and then it generates the damage only during the second time period $-X(1+\delta)^{-1}$; finally, the case in which the disaster does not occur during both time periods (with probability $[1 - F(e_1)]\,[1 - F(e_1, e_2)]$).

3 Uncertain Threshold

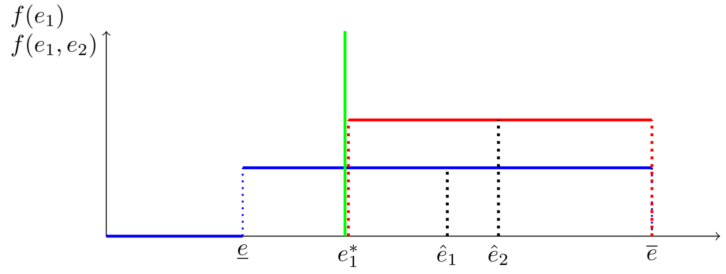
In this section we focus on the random link between emissions and the probability of a natural disaster to take place. We assume the simplest possible distribution function for the critical random level (label it e^c) of the emissions threshold, i.e. we assume that e^c is uniformly distributed and, more specifically, $e^c \sim U(\underline{e}, \overline{e})$.

If $e^c < e_1$, then the environmental disaster takes place during the first time period, whereas in the case $e^c \geqslant e_1$, the environment is preserved (at least, during the first period). Hence, the probability of having an environmental disaster during the first period is given by $Pr\,[e^c < e_1] = F(e_1)$. If the environmental disaster does not take place in the first period, and if we label e_1^* as the optimal emissions in the same period, then the distribution of the uncertain threshold is updated between the first and the second time period, so that the updated distribution in the same period for e^c is such that $e^c \sim U(e_1^*, \overline{e})$. In other words, the emissions produced during the first period become the lower-bound of the uniform random threshold during the second time period. The updating taking place between the first and the second period is due to the fact that the environmental authority learns that the threshold is not in the interval $(\underline{e}, e_1^*)$ and, consequently, any emissions in the interval disappear from the random variable domain. The specific feature of this (simple) learning process is that it is more effective the larger the emissions level in the first period. For this reason, we label it *"dangerous learning"*.

Using a graphical approach in order to present the random variable, we will have a PDF during the second period which is thicker because the variance has decreased (Figure 1). Obviously, the probability of each potential (remaining) level of the threshold will increase; also the entire function moves to the right because the lower bound of possible endurable emissions shifts from $\underline{e}$ to e_1^*.

An additional assumption, to improve the realism of the model, is that a decay rate $\beta \in (0, 1)$ exists such that a quota of the pollutant does not contribute to reaching the uncertain threshold in the second period. In other words, there is an absorption rate between the first and the second period, so that at the beginning of the second time period only $(1-\beta)e_1^*$ units of pollution are accumulated into the environment. Thus, during the second time period, the set of emission levels that do not trigger an

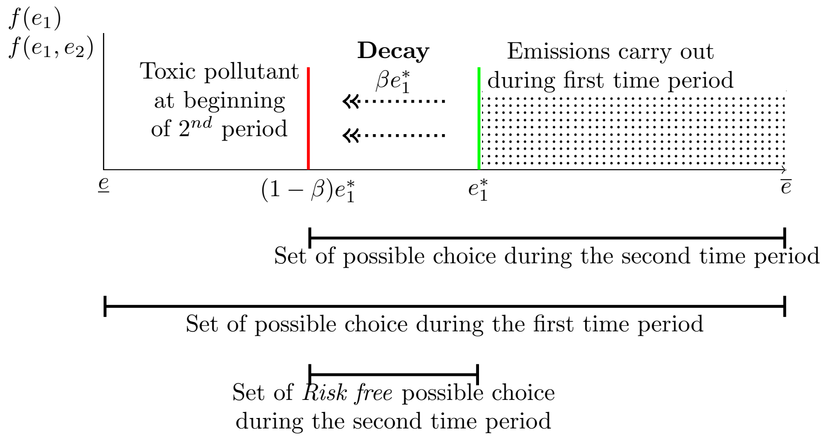
Figure 1: Uncertain Threshold Level



The blue curve shows the PDF during the first period, while the red curve shows the PDF during the second one.

environmental disaster increases by βe_1^*, and these emissions are "free of risk" in the sense that they can be emitted in the second period with a 100% certainty that they will not trigger the disaster. We label these emissions as "risk free emissions" $e_2^{RF} = \beta e_1^*$, which are a sort of "prize" for any risk experienced during the first time period by increasing emissions. This can be expressed in analytical terms as follows:

Figure 2: Emissions' accumulation and decay β



$Pr\left[e^c < (1-\beta)e_1^* + e_2 | e_2 < \beta e_1^*\right] = F(e_1, e_2)_{e_2 < \beta e_1^*} = 0$. The set of "risk free" emission levels described above are as shown in the Figure 2. The dotted area of the figure represents the set of emission levels that are indeed subject to a risk of disaster during the second time period, of course if the disaster itself has not taken place during the first period.

From the above assumptions, we can therefore conclude that the probability to exceed the uncertain threshold, triggering in such a way the disaster, is as follows:

$$F(e_1) = \begin{cases} 0 & for \quad e_1 < \underline{e} \\ \frac{e_1^* - e}{\overline{e} - \underline{e}} & for \quad \underline{e} < e_1 < \overline{e} \\ 1 & for \quad e_1 > \overline{e} \end{cases}$$

and the corresponding density function is given by:

$$f(e_1) = \begin{cases} 0 & for \quad e_1 < \underline{e} \\ \frac{1}{\overline{e}-\underline{e}} & for \quad \underline{e} < e_1 < \overline{e} \\ 0 & for \quad e_1 > \overline{e} \end{cases}$$

This structure is exactly the same used by Barrett (2013). On the other hand, we focus on a two time horizon, so that the first period emissions and the decay rate affect the random variable structure in the second period: as already outlined, the emissions carried out during the first period, e_1^*, become the new lower bound of the domain of the random threshold level. Hence, the probability to trigger a natural disaster during the second time period is influenced by the emissions carried out during the first time period: $F(e_1, e_2) = Pr\,[e^c < (1-\beta)e_1 + e_2 | e^c > e_1^*]$. As a result, during the second time period the random variable is characterized by the following Cdf:

$$F(e_1, e_2) = \begin{cases} 0 & for \quad e_2 < e_1^* \\ \frac{e_2 - \beta e_1^*}{\overline{e}-e_1^*} & for \quad e_1^* < e_2 < \overline{e} \\ 1 & for \quad e_2 > \overline{e} \end{cases}$$

and by the following Pdf:

$$f(e_1, e_2) = \begin{cases} 0 & for \quad e_2 < e_1^* \\ \frac{1}{\overline{e}-e_1^*} & for \quad e_1^* < e_2 < \overline{e} \\ 0 & for \quad e_2 > \overline{e} \end{cases}$$

4 Solving the Model

4.1 Myopic Behaviour

As already outlined, to keep results as neat as possible we focus on the simplest possible shape for the benefits and environmental damages related to emissions, namely, we assume fixed environmental disaster costs X and linear benefits $B(e_i) = e_i$. Under a myopic behaviour, the environmental authority maximizes net expected welfare in each period.

First Time Period

According to the assumptions previously described, the first period objective function is given by:

$$\begin{aligned} &\max_{e_1}\left\{ \left[\frac{\overline{e}-e_1}{\overline{e}-\underline{e}}\right] e_1 + \left[\frac{e_1-\underline{e}}{\overline{e}-\underline{e}}\right][-X] \right\} \\ &s.t: e_1 \geq 0 \end{aligned} \tag{1}$$

The corresponding Lagrangian function is as follows:

$L: \left[\frac{\overline{e}-e_1}{\overline{e}-\underline{e}}\right] e_1 + \left[\frac{e_1-\underline{e}}{\overline{e}-\underline{e}}\right][-X] + \lambda_1 e_1$

First-Order Conditions require:

i) $\left[-\frac{e_1}{\overline{e}-\underline{e}} + \frac{\overline{e}-e_1}{\overline{e}-\underline{e}} - \frac{X}{\overline{e}-\underline{e}} + \lambda_1\right] e_1 = 0$ with $-\frac{e_1}{\overline{e}-\underline{e}} + \frac{\overline{e}-e_1}{\overline{e}-\underline{e}} - \frac{X}{\overline{e}-\underline{e}} + \lambda_1 \leq 0$ and $e_1 \geq 0$

ii) $\lambda_1 e_1 = 0$ with $\lambda_1 \geq 0$ and $e_1 \geq 0$

As a result, we can have an interior solution, featuring $\lambda_1 = 0$ and $e_1^M = \frac{\overline{e}-X}{2} > 0$, and a corner solution, featuring $\lambda_1 = \frac{X-\overline{e}}{\overline{e}-\underline{e}} > 0$ and $e_1^M = 0$. Thus, the emissions' choice depends on the random variable's parameter $\overline{e}$ and on the possible social loss X.

Second Time Period

Under myopic behaviour, the objective function of the environmental authority in the second period is the same as in the first one; the only (indeed relevant) difference is the updating in the random variable:

$$\begin{aligned} &\max_{e_2}\left\{ \left[\frac{\overline{e}-(1-\beta)e_1-e_2}{\overline{e}-e_1}\right] e_2 + \left[\frac{e_2-\beta e_1}{\overline{e}-e_1}\right][-X] \right\} \\ &s.t: e_2 \geq 0 \end{aligned} \tag{2}$$

The Lagrangian function is given by:

$L: \left[\frac{\overline{e}-(1-\beta)e_1-e_2}{\overline{e}-e_1}\right] e_2 + \left[\frac{e_2-\beta e_1}{\overline{e}-e_1}\right][-X] + \lambda_2 e_2$

Along the same lines of the first stage, the solution to the above problem implies, after some straightforward calculations, an interior solution, such that $\lambda_2 = 0$ and $e_2^M = (1+\beta)\left[\frac{\overline{e}-X}{4}\right] > 0$, and a corner solution such that $\lambda_2 = -\frac{\overline{e}-(1-\beta)e_1-X}{\overline{e}-e_1} > 0$, and $e_2^M = 0$.

As a result, focusing on interior solutions, $e_2^M < e_1^M$, i.e. during the second time period emissions are lower than in the first period[1].

This is so because the probability of experiencing a natural disaster raises from the first to the second time period, as we have shown in the previous section (see, again, Equation 1).

It is also interesting to investigate a bit further the role of what we have labelled as "dangerous learning", strictly linked with the decay rate β. In the extreme case where no pollution carries over from

[1]Indeed, it is easily shown that $e_1^M > e_2^M \; : \; \frac{\overline{e}-X}{2} > (1+\beta)\left[\frac{\overline{e}-X}{4}\right]$ if $1 > \beta$, which is true by assumption.

the first to the second time period (i.e. $\beta = 1$), then the environmental authority will be able to set an emission target larger than e_1^M in the second period featuring perfect certainty that the first e_1^M emissions will not trigger any disaster. More broadly, the larger the decay rate, the more "fruitful" is learning, the larger the emissions level will be in the second time period[2].

Finally, notice that, as expected, the decay rate β does not affect the amount of emissions during the first time period: $\frac{\partial}{\partial \beta} e_1^M = 0$. This is the main consequence of the assumed social planner myopia.

4.2 Non-Myopic Behaviour

As argued above, a forward-looking social planner choose emissions levels of both time periods to maximize the expected net present value of social welfare. As a result, the forward looking social planner will indeed consider the impact of the chosen first period emissions on the structure of the random variable during the second time period. Also, and clearly, the non myopic planner anticipates that, if the disaster takes place in the first period, it will affect both time periods. The social planner's objective function in the forward looking scenario is therefore given by:

$$\begin{aligned} \max_{e_1, e_2} \Bigg\{ & \left[\frac{\overline{e} - e_1}{\overline{e} - \underline{e}}\right] e_1 + \left[\frac{e_1 - \underline{e}}{\overline{e} - \underline{e}}\right] \left[-X - X(1+\delta)^{-1}\right] + \left[\frac{\overline{e} - (1-\beta)e_1 - e_2}{\overline{e} - \underline{e}}\right] e_2 (1+\delta)^{-1} \\ & + \left[\frac{e_2 - \beta e_1}{\overline{e} - \underline{e}}\right] \left[-X(1+\delta)^{-1}\right] \Bigg\} \\ & s.t: e_2 \geq 0 \\ & \qquad e_1 \geq 0 \end{aligned} \tag{3}$$

The Lagrangian for this constrained maximization problem is as follows:
$L: \left[\frac{\overline{e}-e_1}{\overline{e}-\underline{e}}\right] e_1 + \left[\frac{e_1-\underline{e}}{\overline{e}-\underline{e}}\right] \left[-X - X(1+\delta)^{-1}\right] + \left[\frac{\overline{e}-(1-\beta)e_1-e_2}{\overline{e}-\underline{e}}\right] e_2 (1+\delta)^{-1} + \left[\frac{e_2-\beta e_1}{\overline{e}-\underline{e}}\right] \left[-X(1+\delta)^{-1}\right] + \lambda_1 e_1 + \lambda_2 e_2$

First-Order Conditions require:

i) $\left[-\frac{e_1}{\overline{e}-\underline{e}} + \frac{\overline{e}-e_1}{\overline{e}-\underline{e}} - \frac{X\left[1+(1+\delta)^{-1}(1-\beta)\right]}{\overline{e}-\underline{e}} - \frac{e_2(1+\delta)^{-1}(1-\beta)}{\overline{e}-\underline{e}} + \lambda_1\right] e_1 = 0$ with $[\cdot] \leq 0$ and $e_1 \geq 0$

ii) $\left[-\frac{e_2(1+\delta)^{-1}}{\overline{e}-\underline{e}} + \frac{\overline{e}-(1-\beta)e_1-e_2}{\overline{e}-\underline{e}}(1+\delta)^{-1} - \frac{X(1+\delta)^{-1}}{\overline{e}-\underline{e}} + \lambda_2\right] e_2 = 0$ with $[\cdot] \leq 0$ and $e_2 \geq 0$

iii) $\lambda_1 e_1 = 0$ with $\lambda_1 \geq 0$ and $e_1 \geq 0$

iv) $\lambda_2 e_2 = 0$ with $\lambda_2 \geq 0$ and $e_2 \geq 0$

After some tedious but straightforward calculations, we can conclude that, focusing on the interior solutions[3]: $e_1^{NM} = \frac{\left[2-(1+\delta)^{-1}(1-\beta)\right]\overline{e} - \left[2+(1+\delta)^{-1}(1-\beta)\right]X}{4-(1+\delta)^{-1}(1-\beta)^2}$, and $e_2^{NM} = \frac{(1+\beta)[\overline{e}-X]+X(1+\delta)^{-1}(1-\beta)}{4-(1+\delta)^{-1}(1-\beta)^2}$.

The main difference with respect to the myopic behaviour is that here, the social planner "shifts"

[2]Straightforward, $\frac{\partial}{\partial \beta} e_2^M = \frac{\overline{e}-X}{4} > 0$ for the interior solution.

[3]corner solutions will be discussed in the next subsection.

emissions from the first to the second time period. In other words, we obtain a result which is reversed with respect to the myopic case, namely[4] $e_1^{NM} < e_2^{NM}$.

4.3 Corner Solutions

Before moving on the comparison of the results obtained in the myopic and forward looking scenario, we will devote this subsection to a discussion of the corner solution arising in the forward looking case, and to their comparison with those arising in a myopic setting.

With myopic preferences, the corner solution reflects the scenario in which the social planner decides not to emit, namely $e_{1,2}^{M} = 0$. Mathematically, this occurs if $X > \overline{e}$: in this scenario, the social planner prefers to stop production because she is not willing to bear the risk of a potentially very damaging disaster (see Figure 3). Thus, if a corner solution arises in the first time period, i.e. $X > \overline{e}$ then it is also $e_2^{M} = 0$. On the other hand, if there are positive emissions during the first time period, then emissions are positive also in the second: $e_{1,2}^{M} > 0 \quad if \quad X < \overline{e}$. This could be expected, as learning implies that some "risk free" emissions can be exploited during the second period.

The impossibility of a scenario in which null second period emissions and positive ones during the first period arise (both in the myopic and non myopic case) can be derived mathematically: such a corner solution would imply:

$$\begin{cases} e_1^{NM} = \frac{\overline{e} - X\left[1+(1+\delta)^{-1}(1-\beta)\right]}{2} \\ e_2^{NM} = 0 \\ \lambda_2 = -\left[\frac{(1+\beta)\overline{e}+\left[2+\beta-(1+\delta)^{-1}(1-\beta)^2\right]X}{\overline{e}-\underline{e}}\right](1+\delta)^{-1} \end{cases}$$

but it is straightforward to show that $\lambda_2 \ngeq 0$, which violates the Khun-Tucker conditions. It is on the other hand possible to have parameters values such that (in a forward-looking setting) positive emissions arise during the second time period while emissions are set to 0 in the first one. Such a solution would indeed imply the following conditions:

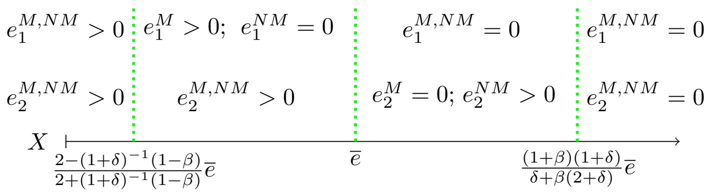
$$\begin{cases} e_1^{NM} = 0 \\ e_2^{NM} = \frac{\overline{e}-X}{2} \\ \lambda_1 = -\frac{\left[2-(1+\delta)^{-1}(1-\beta)\right]\overline{e}-\left[2+(1+\delta)^{-1}(1-\beta)\right]X}{2(\overline{e}-\underline{e})} \end{cases}$$

[4] $e_1^{NM} < e_2^{NM}$ if $\left[2-(1+\delta)^{-1}(1-\beta)\right]\overline{e} - \left[2+(1+\delta)^{-1}(1-\beta)\right]X < (1+\beta)\left[\overline{e}-X\right] + X(1+\delta)^{-1}(1-\beta)$, which requires $-\left[\beta+(1+\delta)^{-1}(1-\beta)\right]\overline{e} - (1-\beta)X < 0$, that is always true.

In the non-myopic case, the social planner takes into consideration that a natural disaster taking place in the first period would also generate damages also in the second period, incurring in loss with net present value $-X - X(1+\delta)^{-1}$ instead of $-X$ arising in the myopic case. As a consequence, the threshold for the social loss X taking the forward looking social planner to stop emissions in the first period is lower than in the myopic case (see Figure 3). Production will not stop also in period 2 only if the value of X is sufficiently small. Summing up, production will stop in the first period but there will be positive emissions in the second if $\frac{2-(1+\delta)^{-1}(1-\beta)}{2+(1+\delta)^{-1}(1-\beta)}\overline{e} < X < \frac{(1+\beta)(1+\delta)}{\delta+\beta(2+\delta)}\overline{e}$, and then $e_2^{NM} > 0$, with $e_1^{NM} = 0$.

Finally, if $X > \overline{e}$ then the social planner will simply stop emissions and production, due to the very

Figure 3: Corner solutions



large magnitude of the possible natural disaster, as we could expect. More specifically, we will have in this case:

$$\begin{cases} e_1^{NM} = 0 \\ e_2^{NM} = 0 \\ \lambda_1 = \frac{X\left[1+(1+\delta)^{-1}(1-\beta)\right]-\overline{e}}{\overline{e}-\underline{e}} \\ \lambda_2 = \frac{X-\overline{e}}{\overline{e}-\underline{e}}(1+\delta)^{-1} \end{cases}$$

5 Myopic vs. Non-Myopic Behaviour

We now turn to compare the results under the two assumed scenarios, focusing on interior solutions. We can easily conclude that the myopic social planner chooses to emit more than the forward looking one in the first period, while the reverse holds in the second period. In fact, we obtain that[5] $e_1^M > e_1^{NM}$, while[6] $e_2^M < e_2^{NM}$. Also, we can show that overall accumulated emissions are larger under a myopic preference structure: $e_2^M + (1-\beta)e_1^M > e_2^{NM} + (1-\beta)e_1^{NM}$. According to these results the probability to experience a disaster is larger in a myopic setting only during the first time period, whereas, the probability to experiment the natural disaster, with a forward-looking behaviour is larger during the second time

[5] $e_1^M > e_1^{NM}$ requires $\frac{\overline{e}-X}{2} > \frac{\left[2-(1+\delta)^{-1}(1-\beta)\right]\overline{e}-\left[2+(1+\delta)^{-1}(1-\beta)\right]X}{4-(1+\delta)^{-1}(1-\beta)^2}$, which holds if $(1+\delta)^{-1}(1-\beta)\left[(1+\beta)\overline{e}-(\beta-3)X\right] > 0$. This is always the case, as $(1+\delta)^{-1}(1-\beta) > 0$ by assumption, and $(1+\beta)\overline{e}-(\beta-3)X > 0$ because $0 < \beta < 1$.

[6] $e_2^M < e_2^{NM}$ requires $(1+\beta)\left[\frac{\overline{e}-X}{2}\right] < \frac{(1+\beta)[\overline{e}-X]+X(1+\delta)^{-1}(1-\beta)}{4-(1+\delta)^{-1}(1-\beta)^2}$ which can be easily shown to hold, as $0 < X(1+\delta)^{-1}(1-\beta)$.

period. Albeit most of these results could be expected, their driving forces (at least in part) rest on (to our knowledge) underinvestigated mechanics.

In order to help the interpretation of results, we provide a numerical example; adopted parameter values are as shown in Table 1, while the corresponding equilibrium values are as reported in Table 2

Table 1: Numerical Example: Parameters

Parameter name	Description	Value
X	Social Cost of an eventual natural disaster	10
$\overline{e}$	Upper-bound of toxic emissions	30
$\underline{e}$	Lower-bound of toxic emissions	0
δ	Interest rate	0.05
β	Decay rate and learning parameter	0.1

Table 2: Numerical Example: Results

e_1^M	e_2^M	$e_2^M + (1-\beta)e_1^M$	$F(e_1^M)$	$F(e_1^M, e_2^M)$	$\left[1 - F_1^M\right]\left[F_2^M\right]$
10	5.5	14.95	0.333	0.225	0.15
e_1^{NM}	e_2^{NM}	$e_2^{NM} + (1-\beta)e_1^{NM}$	$F(e_1^{NM})$	$F(e_1^{NM}, e_2^{NM})$	$\left[1 - F_1^{NM}\right]\left[F_2^{NM}\right]$
1.77	9.203	10.796	0.059	0.32	0.301

As already outlined, the probability of experimenting a natural disaster in the second time period is lower under myopic preference, as compared with the forward-looking preference structure. This is due to the fact that the myopic agent takes "advantage" of her short sightedness during the first time period, exploiting the "dangerous learning" during the second one. In others words, the myopic planner, not accounting for the second period impact of its choices, finds it convenient to trample on the border of the cliff because only in this way she can get closer to discover where the "border" is.

We conclude this section by calculating different possible social welfare calculations, related to each period as well as to aggregate figures. Under a myopic structure: $WB\left[e_1\right] = \left[1 - F(e_1)\right] e_1 - F(e_1)X$, and $WB\left[e_2\right] = \left[1 - F(e_1, e_2)\right] e_2 - F(e_1, e_2)X$. It is the easily shown that $WB\left[e_1^M\right] + (1+\delta)^{-1} WB\left[e_2^M\right] > WB\left[e_1^{NM}\right] + (1+\delta)^{-1} WB\left[e_2^{NM}\right]$. On the other hand, under a forward-looking structure, we obtain the opposite result: $WB\left[e_1^{NM}, e_2^{NM}\right] > WB\left[e_1^M, e_2^M\right]$, where $WB\left[e_1, e_2\right] = \left[1 - F(e_1)\right] e_1 + \left[1 - F(e_1)\right] [1 - F(e_1, e_2)] e_2 - F(e_1) \left[X\left[1 + (1+\delta)^{-1}\right]\right] - \left[1 - F(e_1)\right] F(e_1, e_2) X (1+\delta)^{-1}$. Full welfare comparisons are reported in Table 3.

Table 3: Welfare Analysis

$WB\left[e_1^M\right]$	$WB\left[e_2^M\right]$	$WB\left[e_1^M\right]+(1+\delta)^{-1}WB\left[e_2^M\right]$	$WB\left[e_1^M,e_2^M\right]$
3.333	2.012	5.249	1.572
$WB\left[e_1^{NM}\right]$	$WB\left[e_2^{NM}\right]$	$WB\left[e_1^{NM}\right]+(1+\delta)^{-1}WB\left[e_2^{NM}\right]$	$WB\left[e_1^{NM},e_2^{NM}\right]$
1.075	3.003	3.805	3.641

6 Comparative Statics

The aim of this section is to use the numerical simulation to investigate the impact of changes in the main parameters values on the optimal emissions level, on the probability to trigger a natural disaster and on the welfare level under both preferences structure.

We first focus on the social Cost X of the possible natural disaster (first column of Table 4). Under a myopic preference structure, emissions decrease with X in the first time period, as: $\frac{\partial}{\partial X}e_1^M = -\frac{1}{2} < 0$. In the second time period (yet with myopic behaviour), we get a weaker relation with respect to the first time period: $\frac{\partial}{\partial X}e_2^M = -\frac{(1+\beta)}{4}$. Notice also that the larger is β, the larger is the reactivity of e_2^M to X; indeed, a larger decay rate implies that more "risk free" emissions are available in the second period (if no disaster took place in the first period) as a larger decay rate implies larger second period emissions *ceteris paribus* and, making therefore increases in X more dangerous.

The evidence in the forward-looking case is similar. In fact, there is a negative relation between X and emissions with a non-myopic planner in both time periods: $\frac{\partial}{\partial X}e_1^{NM} : -\frac{2+(1+\delta)^{-1}(1-\beta)}{4-(1+\delta)^{-1}(1-\beta)} < 0$, and $\frac{\partial}{\partial X}e_2^{NM} : \frac{-(1+\beta)+(1+\delta)^{-1}(1-\beta)}{4-(1+\delta)^{-1}(1-\beta)} < 0$.

We can therefore conclude that the social loss related to a possible disaster affects negatively emission levels in both scenarios. Comparing the reactivity of emissions to X in the myopic and non-myopic case, we do not find huge differences. It is however easily shown that $|\frac{\partial}{\partial X}e_1^M| < |\frac{\partial}{\partial X}e_1^{NM}|$; for example, for $X = 20$ and in the forward-looking case, the social planner decides to stop emissions during the first time period, whereas, the myopic planner chooses positive emissions in both periods (Figure 3).

Turning to the impact of changes in the decay rate and, as a result, in the learning opportunities, as measured by β, under myopic preferences, a change in β does not affect first time period emissions (second column of the first panel of the Table 5), as $\frac{\partial}{\partial \beta}e_1^M = 0$; indeed, during the first time period the only reason that pushes the myopic social planner not to increase emissions as much as possible is the fear of triggering the natural disaster. The "dangerous" updating is then exploited by the social planner in the second period. Clearly, the larger is β the larger will be the "risk free" emissions to be exploited in the second period, and, therefore, the larger will be second period emissions themselves: $\frac{\partial}{\partial \beta}e_2^M : \frac{\bar{e}-X}{4} > 0$. In other words, myopic preferences and the related "recklessness" are more "fruitful" if β is larger.

Table 4: Comparative Statics: Social Cost X of possible Natural Disaster

X	e_1^M	e_2^M	$e_2^M + (1-\beta)e_1^M$	$F(e_1^M)$	$F(e_1^M, e_2^M)$	$\left[1 - F_1^M\right]\left[F_2^M\right]$
1	14.5	7.975	21.677	0.483	0.421	0.217
5	12.5	6.875	18.687	0.417	0.321	0.187
10	10	5.5	14.5	0.333	0.225	0.15
20	5	2.75	7.475	0.167	0.09	0.075
30	-	-	-	-	-	

X	$W\left[e_1^M\right]$	$W\left[e_2^M\right]$	$W\left[e_1^M\right] + (1+\delta)^{-1} W\left[e_2^M\right]$	$W\left[e_1^M, e_2^M\right]$
1	7.008	3.777	10.442	8.542
5	5.208	2.634	7.602	4.945
10	3.333	2.012	5.249	1.572
20	0.833	0.475	1.265	-1.65
30	-	-	-	-

X	e_1^{NM}	e_2^{NM}	$e_2^{NM} + (1-\beta)e_1^{NM}$	$F(e_1^{NM})$	$F(e_1^{NM}, e_2^{NM})$	$\left[1 - F_1^{NM}\right]\left[F_2^{NM}\right]$
1	9.734	10.119	18.88	0.324	0.451	0.305
5	6.195	9.712	14.936	0.206	0.382	0.303
10	1.77	9.203	10.796	0.059	0.32	0.301
20	-	5	5	-	0.167	0.167
30	-	-	-	-	-	-

X	$W\left[e_1^{NM}\right]$	$W\left[e_2^{NM}\right]$	$W\left[e_1^{NM}\right] + (1+\delta)^{-1} W\left[e_2^{NM}\right]$	$W\left[e_1^{NM}, e_2^{NM}\right]$
1	6.251	4.834	10.645	9.251
5	3.883	3.901	7.43	6.183
10	1.075	3.003	3.805	3.641
20	-	0.833	0.793	1.136
30	-	-	-	-

Under forward-looking preferences (third and fourth panels of Table 5), the regulator plans to exploit the new information that becomes available between the first and the the second time period, distributing the risk of a natural disaster along the two time periods. Thus, the forward-looking social planner anticipates the policy, postponing the risk to the period when more information will be available.

7 Stochastic decay rate β

In this section, we add another source of uncertainty, introducing a stochastic decay rate. More specifically, we assume that in the first period the social planner does not know whether the decay rate is β_H or β_L, with $\beta_H > \beta_L$, but just at the beginning of the second time period such information is revealed. On the other hand, in the first period the social planner has an *a priori* distribution concerning the parameter β, namely $\beta = \beta_H$ with probability p_H and $\beta = \beta_L$ with probability $(1 - p_H)$. As a result, the expected value of β is $\widehat{\beta} = p_H \beta_H + (1 - p_H)\beta_L$.

Clearly, the myopic social planner is not affected by this new kind of uncertainty, as the value of β affects myopic decision making only in the second period (second column of first panel of Table 6).

On the other hand, during the second time period there is a positive relationship between the (ex post) decay rate and emissions $\frac{\partial}{\partial\beta}e_2^M = \frac{\bar{e}-X}{4} > 0$ (third column of first panel of Table 5), as in the case of a deterministic decay rate.

In the non-myopic case results are significantly different. First of all, as shown in the comparative statics under a deterministic decay rate, any change in it affects the emissions decision in both periods (second and third column of third panel of Table 5). From section 4.2, and substituting $\beta = \widehat{\beta}$, emissions under a non-myopic social planner are in this case:

$$\begin{cases} e_1^{NM} = \frac{\left[2-(1+\delta)^{-1}(1-\widehat{\beta})\right]\bar{e}-\left[2+(1+\delta)^{-1}(1-\widehat{\beta})\right]X}{4-(1+\delta)^{-1}(1-\widehat{\beta})^2} \\ e_2^{NM} = \frac{(1+\widehat{\beta})[\bar{e}-X]+X(1+\delta)^{-1}(1-\widehat{\beta})}{4-(1+\delta)^{-1}(1-\widehat{\beta})^2} \end{cases}$$

As a result, any change in the a priori distribution or in the ex post values of β that lead to a change in $\widehat{\beta}$, affects emissions level during both periods. More specifically, we can derive comparative statics results regarding probability p_H that the social planner links to having a high decay rate, β_H. In order to cover all possible scenarios, we chose "extreme" values of the high and low ex post decay rates: $\beta_H = 0,9$ and $\beta_L = 0,1$. Thus, if $\beta = \beta_H$ the environment is able to absorb most emissions produced in the first time period.

Comparing the myopic and non-myopic cases, according to the numerical example provided, we notice that emissions carried out during the second time period under myopic case, when $\beta = \beta_L$ ex post, are lower than the ones carried out in the forward-looking case; indeed, for any value of p_H: $e_2^M(\beta_L) < e_2^{NM}$. On the other hand (Table 6), when $\beta = \beta_H$ ex post, the reverse holds true: $e_2^M(\beta_H) > e_2^{NM} \quad \forall \quad p_H$. Hence: $e_2^M(\beta_H) > e_2^{NM}(p_H) > e_2^M(\beta_L) \quad \forall \quad p_H$. Thus, we can argue that the forward-looking social planner spreads the risk along the two time periods, but she does not exploit the new information on the realization of β that is revealed between the first and the second period. On the other hand, the myopic social planner takes most of the risk during the first time period but then she exploits the new information during the second one.

As a result, the same conclusions apply when comparing overall emissions: $(1-\beta_H)e_1^M + e_2^M(\beta_H) > (1-\widehat{\beta})e_1^{NM} + e_2^{NM}(\widehat{\beta}) > (1-\beta_L)e_1^M + e_2^M(\beta_L) \quad \forall \quad p_H$. Myopic behaviour allows the planner to exploit all the information that is revealed across the two time periods, but also implies a larger risk run in the first time period. Clearly, in case of "bad news" (β_L), the welfare for the myopic social planner dramatically decreases; in this case, the risk taken in the first period does not pay off in the second, as the ex post decay rate is very low. We can note, more generally, that, under a myopic social planner, welfare is decreasing (increasing) over time when $\beta = \beta_L$ ($\beta = \beta_H$) ex post. On the other hand, a forward looking social planner always implies an increasing welfare over time. On the other hand, even in the best possible scenario, the welfare increase obtained in the forward looking case would not mimic that obtained in the myopic case.

8 Discussion and Empirical Extensions

We have investigated, in a setting *a la* Barrett (2013), augmented to account for two time periods, as in Tarui and Polasky (2005), the impact on environmental policy and welfare of uncertainty regarding the possibility of a natural disaster. We have addressed two possible institutional settings, one featuring a myopic social planner and one featuring a forward-looking planner, and obtained that no straightforward welfare ordering is possible, albeit emissions comparisons turned out to lead to expected outcomes. An important role in our results is played by what we called "dangerous learning", i.e. the incentive of the myopic planner to push emissions further than the forward looking one, learning in such a way more information on where the actual threshold is, of course at the expenses of a larger risk of disaster in the first period. Clearly, it is more important for the myopic social planner to have a large decay rate. These considerations imply that, the comparison between myopic and forward looking settings turn out not to be straightforward: the countervailing forces of risk spreading over time vs. information updating benefits may lead to non-myopic paradox, where welfare in the second period is larger in a myopic setting if the myopic planner pushes emissions far enough in the first period. Also, by introducing a stochastic decay rate, we show that the environment can punish or reward myopic behaviour, depending on its ex post value.

Our future research will be devoted to test empirically the conclusions of our theoretical investigation, with a focus on the existence of a "dangerous learning" effect. We aim at performing this very challenging task by estimating the potential link between measures of unsustainable land use and floods or landslide events in the Italian territory. Moreover, we are interested in investigating whether changes in land use (for example from agriculture to residential) or natural events, such as rainfall, have in the past triggered changes in uncertainty, as measured by the perceived probability of catastrophic events.

Table 5: Comparative Statics: Decay Rate β of toxic emissions

β	e_1^M	e_2^M	$e_2^M + (1-\beta)e_1^M$	$F(e_1^M)$	$F(e_1^M, e_2^M)$	$\left[1-F_1^M\right]\left[F_2^M\right]$
0	10	5	15	0.333	0.25	0.167
0.1	10	5.5	14.5	0.333	0.225	0.15
0.2	10	6	14	0.333	0.2	0.133
0.3	10	6.5	13.5	0.333	0.175	0.117
0.4	10	7	13	0.333	0.15	0.1
0.5	10	7.5	12.5	0.333	0.125	0.083
0.6	10	8	12	0.333	0.1	0.067
0.7	10	8.5	11.5	0.333	0.075	0.05
0.8	10	9	11	0.333	0.05	0.033
0.9	10	9.5	10.5	0.333	0.025	0.017

β	$W\left[e_1^M\right]$	$W\left[e_2^M\right]$	$W\left[e_1^M\right] + (1+\delta)^{-1}W\left[e_2^M\right]$	$W\left[e_1^M, e_2^M\right]$
0	3.333	1.25	4.524	1.071
0.1	3.333	2.012	5.249	1.572
0.2	3.333	2.8	6	2.089
0.3	3.333	3.612	6.773	2.623
0.4	3.333	4.45	7.571	3.173
0.5	3.333	5.312	8.392	3.74
0.6	3.333	6.2	9.238	4.324
0.7	3.333	7.112	10.107	4.924
0.8	3.333	8.05	11	5.541
0.9	3.333	9.012	11.916	6.175

β	e_1^{NM}	e_2^{NM}	$e_2^{NM} + (1-\beta)e_1^{NM}$	$F(e_1^{NM})$	$F(e_1^{NM}, e_2^{NM})$	$\left[1-F_1^{NM}\right]\left[F_2^{NM}\right]$
0	0.625	9.687	10.312	0.21	0.33	0.323
0.1	1.77	9.203	10.796	0.052	0.32	0.301
0.2	2.809	8.876	11.123	0.094	0.301	0.277
0.3	3.774	8.679	11.321	0.126	0.288	0.252
0.4	4.687	8.594	11.406	0.156	0.265	0.224
0.5	5.57	8.607	11.392	0.186	0.238	0.194
0.6	6.436	8.713	11.287	0.214	0.206	0.162
0.7	7.299	8.905	11.095	0.243	0.167	0.126
0.8	8.173	9.183	10.818	0.272	0.121	0.088
0.9	9.069	9.546	10.453	0.302	0.066	0.046

β	$W\left[e_1^{NM}\right]$	$W\left[e_2^{NM}\right]$	$W\left[e_1^{NM}\right] + (1+\delta)^{-1}W\left[e_2^{NM}\right]$	$W\left[e_1^{NM}, e_2^{NM}\right]$
0	0.404	3.195	3.447	3.487
0.1	1.075	3.003	3.805	3.641
0.2	1.61	3.104	4.566	3.663
0.3	2.041	3.304	5.188	3.851
0.4	2.392	3.658	5.876	4.098
0.5	2.679	4.173	6.653	4.401
0.6	2.91	4.86	7.538	4.761
0.7	3.09	5.744	8.56	5.18
0.8	3.222	6.859	9.754	5.66
0.9	3.304	8.254	11.165	6.206

Table 6: Comparative Statics: Decay Rate $\widehat{\beta}$ of toxic emissions

$\widehat{\beta}$	e_1^M	e_2^M	$e_2^M + (1-\widehat{\beta})e_1^M$	$F(e_1^M)$	$F(e_1^M, e_2^M)$	$\left[1-F_1^M\right]\left[F_2^M\right]$
β_L	10	5.5	14.5	0.333	0.225	0.15
β_H	10	9.5	10.5	0.333	0.025	0.017
$\widehat{\beta}$	$W\left[e_1^M\right]$	$W\left[e_2^M\right]$	$W\left[e_1^M\right] + (1+\delta)^{-1}W\left[e_2^M\right]$		$W\left[e_1^M, e_2^M\right]$	
β_L	3.333	2.012	4.524		1.071	
β_H	3.333	9.012	11.916		6.175	
p_H	e_1^{NL}	e_2^{NL}	$e_2^{NL} + (1-\widehat{\beta})e_1^{NL}$	$F(e_1^{NL})$	$F(e_1^{NL}, e_2^{NL})$	$\left[1-F_1^{NL}\right]\left[F_2^{NL}\right]$
0.1	2.608	8.931	11.07	0.087	0.311	0.284
0.2	3.395	8.744	11.256	0.113	0.295	0.262
0.3	4.144	8.632	11.367	0.138	0.279	0.241
0.4	4.866	8.589	11.411	0.162	0.26	0.218
0.5	5.57	8.607	11.392	0.186	0.238	0.164
0.6	6.263	8.685	12.333	0.21	0.213	0.168
0.7	6.953	8.818	11.182	0.232	0.183	0.141
0.8	7.647	9.006	10.994	0.255	0.15	0.112
0.9	8.35	9.248	10.751	0.278	0.111	0.08
p_H	$W\left[e_1^{NM}\right]$	$W\left[e_2^{NM}\right]$	$W\left[e_1^{NM}\right] + (1+\delta)^{-1}W\left[e_2^{NM}\right]$		$W\left[e_1^{NM}, e_2^M\right]$	
0.1	1.512	3.083	4.448		3.633	
0.2	1.879	3.205	4.931		3.769	
0.3	2.19	3.427	5.454		3.943	
0.4	2.455	3.748	6.024		4.154	
0.5	2.679	4.173	6.653		4.401	
0.6	2.868	4.708	7.352		4.685	
0.7	3.024	5.365	8.133		5.005	
0.8	3.149	6.16	9.016		5.364	
0.9	3.242	7.113	10.016		5.763	

References

[1] Barrett, S. (2013): "Climate Treaties and Approaching Catastrophes", *Journal of Environmental Economics and Management*, Vol. 66, pp. 235–250.

[2] Costello, C. and L. Karp (2004): "Dynamic Taxes and Quotas with Learning", *Journal of Economic Dynamic & Control*, Vol. 28, pp. 1661–1680.

[3] Kurz, M. (1987): *"Myopic Decision Rule" in A Dictionary of Economics*. Palgrawe Macmillan.

[4] Newell, R. and W. Pizer (2003): "Regulating Stock Externalities under Uncertainty", *Journal of Environmental Economics and Management*, Vol. 45, pp. 416–432.

[5] Robert, M. and A. Spence (1976): "Effluent Charges and Licenses under Uncertainty", *Journal of Public Economics*, Vol. 5, No. 3-4, pp. 193–208.

[6] Stavins, R. N. (1996): "Uncertainty and Instrument Choice", *Journal of Environmental Economics and Management*, Vol. 30, pp. 218–232.

[7] Tarui, N. and S. Polasky (2005): "Environmental regulation with technology adoption, learning and strategic behavior", *Journal of Environmental Economics and Management*, Vol. 50, No. 3, pp. 447–467.

[8] Weitzman, M. (1974): "Prices vs. Quantities", *Rev. Econom. Studies*, Vol. 41, pp. 477–491.